Measuring Exponential Decay Slope

Waveform Math Determines Exponential Time Constants

Many physical phenomena result in waveforms with exponential rising and/or falling edges where the exponential time constant reveals information about the underlying process. While it is possible to determine the exponential time constant by direct measurement using cursors it is often far easier and more accurate to logarithmically weight the signal and read the slope of the signal directly.

A typical exponential process can be described by an equation of the form:

$$V(t) = -ae^{-t/t} + b$$

where:
a and b are arbitrary constants
t is the exponential time constant and t ³ 0

The time constant, τ, is the unknown to be determined. A waveform of this type is shown as the upper trace (2) in figure 1.

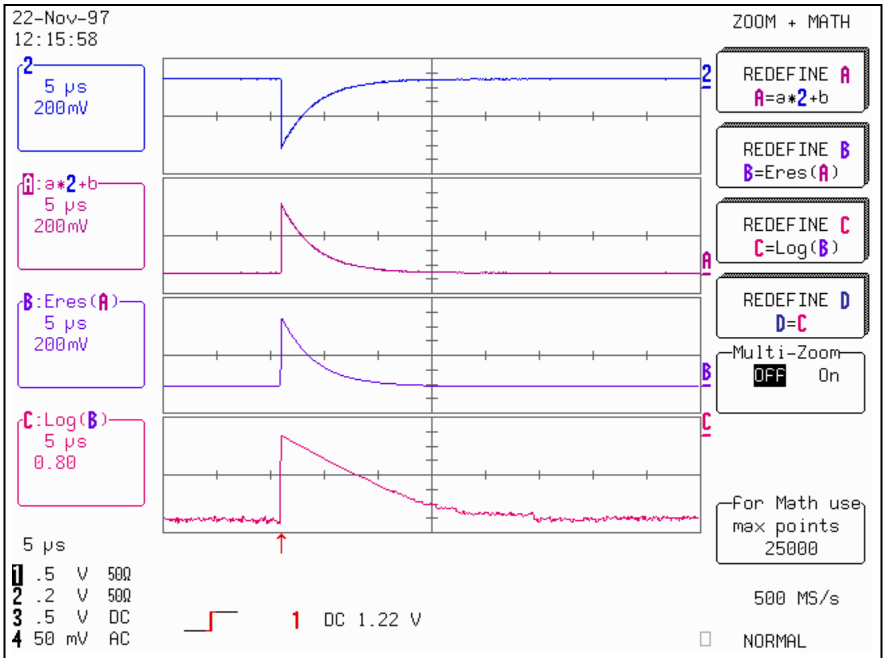

Figure 1- The math setup to linearize an exponential signal

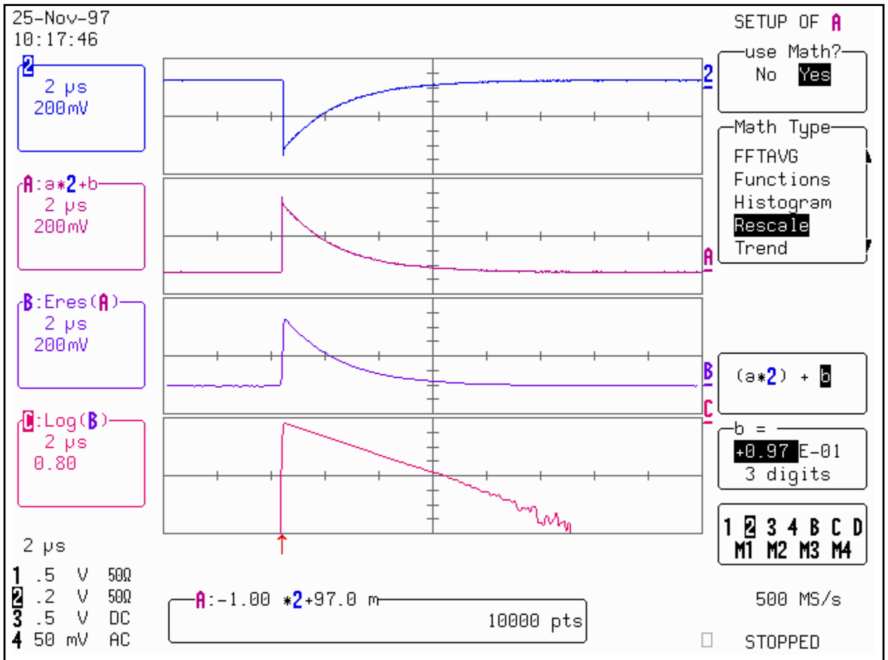

Figure 2 - Setup of the rescale math function to make the waveform unipolar positive.

The next step in determining the time constant is to convert the signal to a positive unipolar waveform prior to taking the natural logarithm. The value of the constants a and b, the gain and a fixed voltage offset, are measured using cursors or parameters. This data is used to rescale the waveform as shown in figure 2 . The waveform is inverted (multiplied by -1) and the offset is removed.

A useful step in the process is to increase the dynamic range of the measurement using the enhanced resolution function. While not strictly required it does improve the measurement accuracy by filtering the signal using a Gaussian low pass digital filter. The enhanced resolution setup is shown in figure 3.

To determine the exponential time constant, **t**, we can take the natural logarithm of both sides of the original exponential expression and solve for τ:

$$\ln [-(V(t)+b)/a] = -t/t$$

$$t = -t / \ln[-(V(t)+b)/a]$$

The measurement setup for determining the slope of the logarithmic expression is shown in figure 4. While relative cursors could be used to determine the voltage and time differences required for calculating the slope the use of parameters permits direct measurement without calculation. Note that the parameter falltime at user specified levels is used. The levels are set up so that the amplitude difference has a value of 1. With this amplitude threshold setup the falltime reads the time constant directly. The slope of the logarithmically weighted signals has the best linearity in the vicinity of the peak. The amplitude thresholds should be set as close to the peak value as possible to obtain the best accuracy.

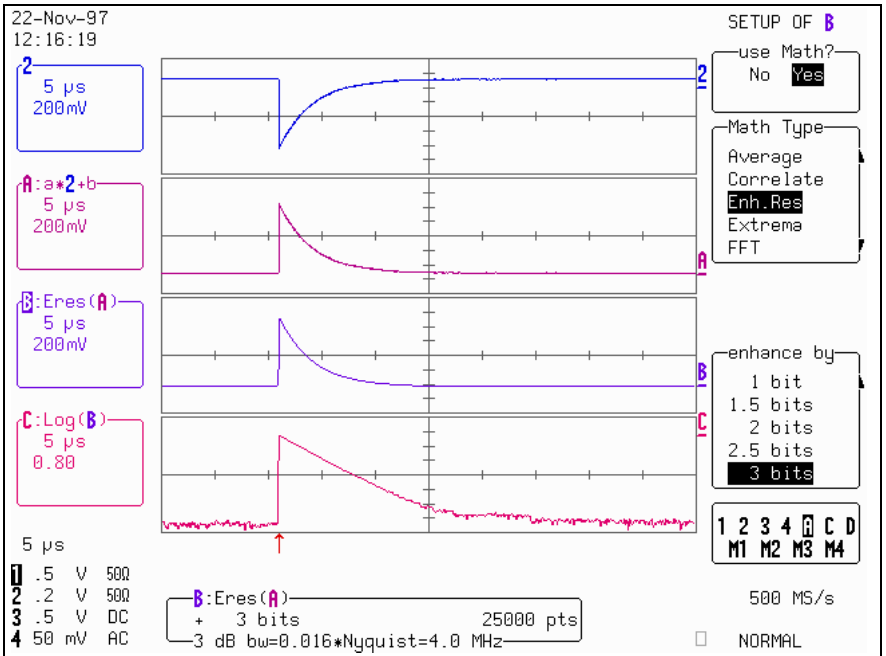

Figure 3 - Improving the dynamic range of the measurement using 3 bits of enhanced resolution in trace B.

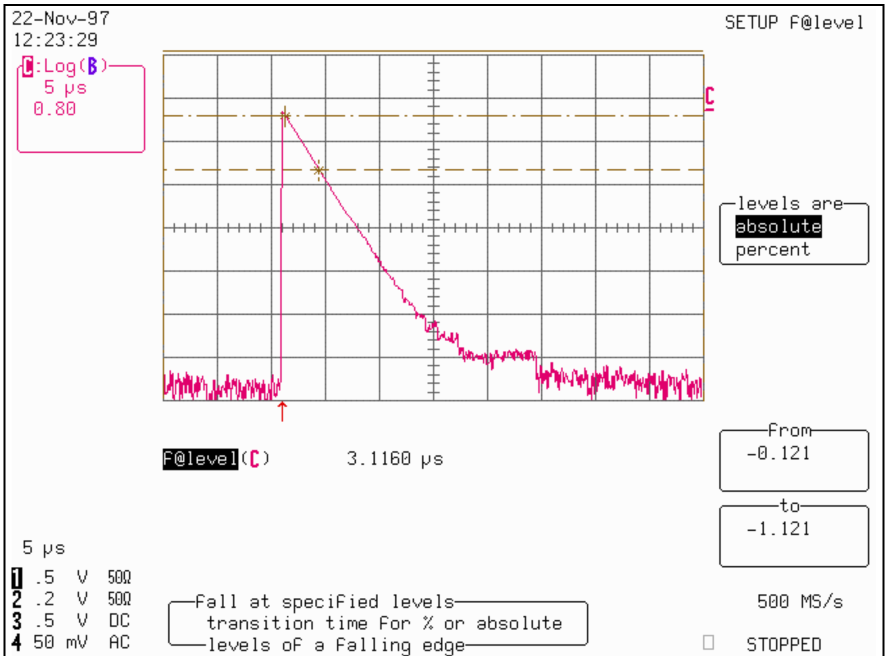

Figure 4 - Setup of the falltime at user specified level to read time constant directly